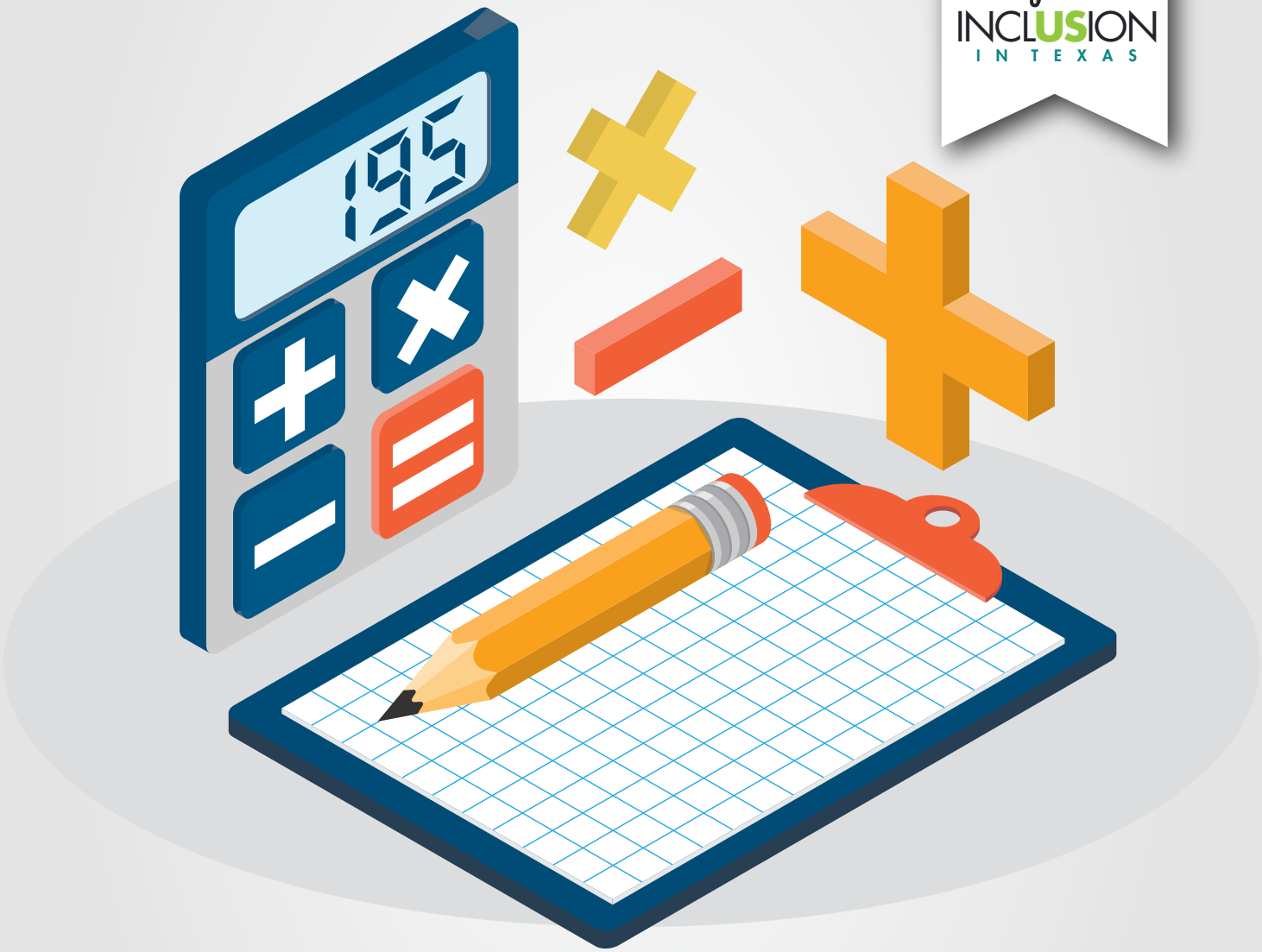
INCLUSION IN TEXAS



Instructional Routines for Mathematics Intervention

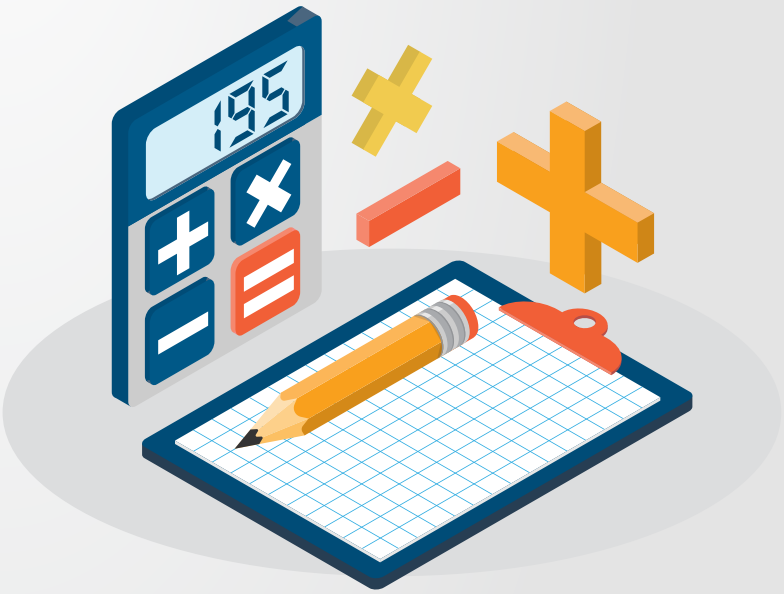
The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 21

Ratios, Proportions, Rates, and Percentages



Module 21: Ratios, Proportions, Rates, and Percentages
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
coefficient	A number that is multiplied by a variable.
constant	A term that does not change; a number on its own.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent fractions	Fractions that have different numerators and denominators that represent the same value or proportion of the whole.
equivalent ratios	Ratios that have the same fractional number, value, or measure.
fraction	A number representing part of a whole or set.
improper fraction	Any fraction in which the numerator is greater than the denominator.
least common multiple	The common multiple with the least value.
like fractions	Fractions that have the same denominator.
lowest terms	A fraction is simplified to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.
mixed number	A whole number and a fraction combined.
multiple	The product of a number and any integer.
numerator	The term in a fraction that tells how many parts of a fraction.
percentage	A rate of an amount per hundred.
proper fraction	A fraction where the numerator is less than the denominator.
proportion	An equation that states that two ratios are equal.
rate	A comparison of two quantities that have different units of measure.
ratio	A comparison of two quantities that have the same unit of measure.
unit rate	A ratio that is written as a number to one.
unlike fractions	Fractions that have different denominators.
variable	A symbol for an unknown value, which is usually represented by a letter.

B. Background Information

In this module, we focus on representing (1) ratios, (2) proportions, (3) rates, and (4) percentages.

C. Routines and Examples

(1) Representing Ratios

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like blocks or shapes

ROUTINE WITH GEOMETRIC SHAPES

Teacher	**Let's show different ratios. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**A ratio is an expression. In a ratio, we compare how much of one amount we have compared to another amount. With ratios, we can compare parts to parts or parts to a whole. How can we compare ratios?**
Students	Parts to parts or parts to a whole.
Teacher	**So, let's show different ratios. We'll use these geometric shapes.** (Show manipulatives.) (Show ratio.)
Teacher	**What's this ratio?**
Students	__ to __.
Teacher	**When we read ratios, make sure to say "to" between the numbers. So, __** (read numbers and emphasize "to")**. Let's say that together.**
Students	__ to __.
Teacher	**Let's show this ratio by comparing parts to parts. What's the first number in the ratio?**
Students	__.
Teacher	**So, let's show __** (first number) **of the shapes. Let's show __ squares. How many?**
Students	__. (Show using shapes.)
Teacher	**Now, what's the second number in the ratio?**
Students	__.
Teacher	**So, let's show __** (second number) **of the shapes. Let's show __ triangles. How many?**

Students __.
(Show using shapes.)

Teacher **With this ratio, __** (first number) **are squares and __** (second number) **are triangles. The ratio of squares to triangles is __ to __. Say that with me.**

Students __ to __.

Teacher **We write our ratio using the colon. I write __ to __ as __** (first number) **colon __** (second number). **Let's write the ratio.**
(Write ratio.)

Teacher **We also can write a ratio as a fraction. The first number in the ratio will be the numerator and the second number will be the denominator. How do we write a ratio as a fraction?**

Students Write the first number as the numerator and second number as the denominator.

Teacher **Let's write this ratio as a fraction.**
(Write fraction.)

Teacher **What's the fraction?**

Students __.

Teacher **If we write a fraction for a part to part ratio, we don't read the fraction as a fraction. We can write it as a fraction but we don't read it as a fraction. Should we read this as a fraction?**

Students No.

Teacher **Now, let's think about the ratio in a different way. Another way to show a ratio is to compare parts to the whole or set. What's another way to show a ratio?**

Students To compare parts to the whole or set.

Teacher **Let's use the squares and triangles from before. Altogether, we have 1, 2, 3…** (count) **shapes. How many squares are in this set?**

Students __.

Teacher **How many shapes are there in the set?**

Students __.

Teacher **If there are __ squares, the ratio of squares to all of the shapes is __ to __. What's the ratio?**

Students __ to __.

Teacher **Let's write that ratio.**
(Write ratio.)

Teacher **Let's write this ratio as a fraction.**
(Write fraction.)

Teacher **What's the fraction?**

Students __.

Teacher **Now, how many triangles are in this set?**

Students __.

Teacher **If there are __ triangles, the ratio of triangles to all of the shapes is __ to __. What's the ratio?**

Students __ to __.

Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	__.
Teacher	**One way to show ratios is to compare parts to parts. How do you show parts to parts?**
Students	Show the number of squares and compare the number of squares to the number of triangles.
Teacher	**When we compare parts to parts, we show two different objects and compare one object, like squares, to another object, like triangles. What other objects could you use to compare parts to parts?**
Students	Cats and dogs, blue cubes and red cubes, cereal and candy.
Teacher	**Another way to show ratios is to compare parts to a whole or the set. How do you compare parts to a set?**
Students	Show the number of squares and compare the number of squares to the number of all of the shapes in a set.
Teacher	**When we compare parts to a whole or a set, we show two different objects and compare one object, like squares, to all of the objects in the set. We use the same objects but we think about the ratio in a different way. Let's review. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**How do you write a ratio as a fraction?**
Students	Write the first number as the numerator and the second number as the denominator.
Teacher	**Great work! Using these objects helps you understand the different ratios. How can you use objects to show a ratio?**
Students	You could show ratios using shapes like squares and triangles. To compare parts to parts, you compare the squares to the triangles. To compare parts to the whole, you compare one shape, like squares, to all of the shapes.

Example

4 : 3

EXAMPLE WITH COLORED CUBES

Teacher	**Let's show different ratios. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**A ratio is an expression. In a ratio, we compare how much of one amount we have compared to another amount. With ratios, we can compare parts to parts or parts to a whole. How can we compare ratios?**
Students	Parts to parts or parts to a whole.

Teacher	**So, let's show different ratios. We'll use these colored cubes.** (Show manipulatives.) (Show ratio.)
Teacher	**What's this ratio?**
Students	4 to 3.
Teacher	**When we read ratios, make sure to say "to" between the numbers. So, 4 *to* 3. Let's say that together.**
Students	4 *to* 3.
Teacher	**Let's show this ratio by comparing parts to parts. What's the first number in the ratio?**
Students	4.
Teacher	**So, let's show 4 of the colored cubes. Let's use the blue cubes. Let's show 4 blue cubes. How many?**
Students	4. (Show using cubes.)
Teacher	**Now, what's the second number in the ratio?**
Students	3.
Teacher	**So, let's show 3 of the colored cubes. Let's use the yellow cubes. Let's show 3 yellow cubes. How many?**
Students	3. (Show using cubes.)
Teacher	**With this ratio, 4 are blue and 3 are yellow. The ratio of blue to yellow is 4 to 3. Say that with me.**
Students	4 to 3.
Teacher	**We write our ratio using the colon. I write 4 to 3 as 4 colon 3. Let's write the ratio.** (Write ratio.)
Teacher	**We also can write a ratio as a fraction. The first number in the ratio will be the numerator and the second number will be the denominator. How do we write a ratio as a fraction?**
Students	Write the first number as the numerator and second number as the denominator.
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{4}{3}$.
Teacher	**If we write a fraction for a part to part ratio, we don't read the fraction as four-thirds. We can write it as a fraction but we don't read it as a fraction. Should we read this as a fraction?**
Students	No.
Teacher	**Now, let's think about the ratio in a different way. Another way to show a ratio is to compare parts to the whole or set. What's another way to show a ratio?**

Students	To compare parts to the whole or set.
Teacher	**Let's use the blue and yellow cubes from before. Altogether, we have 1, 2, 3, 4, 5, 6, 7 cubes. How many blue cubes are in this set?**
Students	4.
Teacher	**And how many cubes are there in the set altogether?**
Students	7.
Teacher	**If there are 4 blue cubes, the ratio of blue cubes to all of the cubes is 4 to 7. What's the ratio?**
Students	4 to 7.
Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{4}{7}$.
Teacher	**We can read this as four-sevenths. How can we read this fraction?**
Students	Four-sevenths.
Teacher	**Now, how many yellow cubes are in this set?**
Students	3.
Teacher	**If there are 3 yellow cubes, the ratio of yellow cubes to all of the cubes is 3 to 7. What's the ratio?**
Students	3 to 7.
Teacher	**Let's write that ratio.** (Write ratio.)
Teacher	**Let's write this ratio as a fraction.** (Write fraction.)
Teacher	**What's the fraction?**
Students	$\frac{3}{7}$.
Teacher	**We can read this as three-sevenths. How can we read this fraction?**
Students	Three-sevenths.
Teacher	**One way to show ratios is to compare parts to parts. How do you show parts to parts?**
Students	Show the number of blue cubes and compare the number of blue cubes to the number of yellow cubes.
Teacher	**When we compare parts to parts, we show two different objects and compare one object, like blue cubes, to another object, like yellow cubes. What other objects could you use to compare parts to parts?**
Students	Cats and dogs, squares and triangles, cereal and candy.
Teacher	**Another way to show ratios is to compare parts to a whole or the set. How do you compare parts to a set?**
Students	Show the number of blue cubes and compare the number of blue cubes to the number of all of the cubes.

Teacher	**When we compare parts to a whole or a set, we show two different objects and compare one object, like blue cubes, to all of the cubes in the set. We use the same objects but we think about the ratio in a different way. Let's review. What's a ratio?**
Students	An expression in which we compare one quantity to another.
Teacher	**How do you write a ratio as a fraction?**
Students	Write the first number as the numerator and the second number as the denominator.
Teacher	**Great work! Using these objects helps you understand the different ratios. How can you use objects to show a ratio?**
Students	You could show ratios using cubes like blue cubes and yellow cubes. To compare parts to parts, you compare the blue cubes to the yellow cubes. To compare parts to the whole, you compare one color, like blue cubes, to all the cubes.

(2) Representing Proportions

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or fraction tiles

ROUTINE WITH FRACTION TILES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Most often, a proportion has an unknown. We use an equation to solve for the unknown within a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.
Teacher	**This proportion does have two fractions. Remember, fractions also can be used to represent the ratios within a proportion. This proportion has an unknown. We will solve for the unknown with these fraction tiles.** (Show manipulatives.)
Teacher	**So, __** (first fraction) **is equal to __** (second fraction)**. Let's read that together.**
Students	__ is equal to __.
Teacher	**Which fraction does not have an unknown?**
Students	__.
Teacher	**Let's show that fraction with the fraction tiles. First, I show the denominator divided into equal parts. Then, we show the numerator with the equal parts.** (Show fraction with fraction tiles.)
Teacher	**Now, which fraction does have an unknown?**
Students	__.
Teacher	**What's unknown – the numerator or the denominator?**
Students	Numerator/denominator.
Teacher	**If the numerator is the unknown, we'll use the denominator to divide another whole into equal parts. We'll learn how many of the numerator parts are equal to the other fraction in our proportion. Let's do an example when the numerator is unknown. I take another whole and divide that whole into the equal parts of the denominator. What's the denominator?**
Students	__.
Teacher	**Let's divide the whole into __ equal parts.** (Show denominator with fraction tiles.)
Teacher	**Now, let's place this fraction near our other fraction so we can compare. I like to place them one above the other. Let's compare the fractions. How many**

	equal parts of this unknown fraction are equivalent to the numerator of the known fraction?
Students	__.
Teacher	**Yes. The numerator would be __. What's the numerator of the unknown fraction?**
Students	__.
Teacher	**Let's write in __ for the unknown.** (Write unknown.)
Teacher	**If the denominator is the unknown, we'll use the numerator and place in numerator parts compared to a whole. Let's do an example when the denominator is unknown. I take another whole. What's the numerator?**
Students	__.
Teacher	**Now, let's place this whole so we can compare it to our other fraction in the proportion. I like to place them one above the other. Let's compare the fractions. How many equal numerator parts of this unknown fraction could be used to be equivalent to the numerator of the know fraction?**
Students	__.
Teacher	**Let's show the numerator with __ equal parts.** (Show numerator with fraction tiles.)
Teacher	**We use one-__**(denominator) **parts to show the numerator. That means the denominator is __. Let's write in __** (denominator) **for the unknown.** (Write unknown.)
Teacher	**Let's read the proportion. __ is equal to __. Let's say that together.**
Students	__ is equal to __.
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**Using objects helps you understand the how to solve for the unknown in a proportion. How can you use objects to solve for an unknown in a proportion?**
Students	You could use fraction tiles to show the known fraction. Then, you could use another set of fraction tiles to compare fractions to determine the unknown in an equivalent fraction.

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Often, a proportion has an unknown, and we use an equation to solve for the unknown within a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.

Teacher	**This proportion does have two fractions. Remember, fractions can be used to represent ratios. This proportion has an unknown. We will solve for the unknown using multiplication and division. Let's read the proportion.**
Students	__ is equal to __.
Teacher	**So, __** (first fraction) **is equal to __** (second fraction)**. Where's the unknown in this proportion?**
Students	__.
Teacher	**We have to determine the unknown in this proportion. The unknown is marked by *x/y/a.* We can determine the unknown by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	__.
Teacher	**The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out.)
Teacher	**Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the second fraction. Let's write __.** (Write.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.

Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. What's the value of the unknown?**
Students	__.
Teacher	**Now, there is another way to solve for an unknown. Where is the unknown in this problem?**
Students	Numerator/denominator.
Teacher	**The unknown is in the numerator/denominator, so look at the denominator/numerator. Look at __** (first denominator/numerator) **and __** (second denominator/numerator)**. What do you notice about the relationship between __ and __?**
Students	(Describes relationship.)
Teacher	**Yes! I see that if you multiply/divide by __ with the first denominator/numerator, that equals the second denominator/numerator. It's like a rule in a function! Let's apply that rule to the numerator/denominator. What are the numerator/denominators in each fraction?**
Students	__/*x* and *x*/__.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Multiply/divide.

Teacher	**Using the same rule as the denominator/numerator, *x* would be __. What's *x*?**
Students	__.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators/denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**What's one way we solved for an unknown in a proportion?**
Students	We multiplied the first denominator by the first numerator and the second numerator. Then, we multiplied the second denominator by the first numerator and the second numerator. Then, we divided by a coefficient to solve for the unknown.
Teacher	**What's another way we solved for an unknown in a proportion?**
Students	We determined the rule of the relationship between the numerators/denominators and applied that rule to determine the unknown.

Example

$$\frac{x}{12} = \frac{2}{3}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's look at a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**A proportion shows two equal ratios. Often, a proportion has an unknown, and we use an equation to solve for the unknown with a proportion. Look at this proportion. What do you notice?** (Show proportion.)
Students	Two fractions.
Teacher	**This proportion does have two fractions. Remember, fractions can be used to represent ratios. This proportion has an unknown. We will solve for the unknown using multiplication and division. Let's read the proportion.**
Students	$\frac{x}{12}$ is equal to $\frac{2}{3}$.
Teacher	**Where's the unknown in this proportion?**
Students	In the first fraction.

Teacher	**We have to determine the unknown in this proportion. The unknown is marked by *x*. We can determine the unknown by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	12.
Teacher	**The denominator of the first fraction is 12. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 12.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 12 and have a denominator of 12, 12 divided by 12 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out 12 and 12.)
Teacher	**Now, multiply the second numerator by 12.** (Write × 12.) **What's the product of 12 times 2?**
Students	24.
Teacher	**We now have a numerator of 24 in the second fraction. Let's write 24.** (Write 24.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by 3.** (Write × 3.) **What's the product of 3 times *x*?**
Students	3*x*.
Teacher	**We now have a numerator of 3*x* in the first fraction. Let's write 3*x*.** (Write 3*x*.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write × 3.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.**

(Cross out 3 and 3.)

Teacher **Using multiplication, we've changed our proportion to the equation 3*x* equals 24. What's the equation?**

Students 3*x* equals 24.

Teacher **Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**

Students It's the constant multiplied by a variable.

Teacher **A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**

Students Divide each side of the equation by the coefficient.

Teacher **What's the coefficient?**

Students 3.

Teacher **Let's divide each side of the equation by 3. Whatever we do to one side of the equal sign we also have to do to the other. What's 24 divided by 3?**

Students 8.

(Write 8.)

Teacher **So, the variable equals 8. What's the value of the unknown?**

Students 8.

Teacher **That's right. $\frac{8}{12}$ is equal to $\frac{2}{3}$. Say that with me.**

Students $\frac{8}{12}$ is equal to $\frac{2}{3}$.

Teacher **Now, there is another way to solve for an unknown. Where is the unknown in this problem?**

Students Numerator.

Teacher **The unknown is in the numerator, so look at the denominators. Look at 12 and 3. What do you notice about the relationship between 12 and 3?**

Students If you divide 12 by 4, you get 3.

Teacher **Yes! I see that if you divide by 4, 12 divided by 4 equals the second denominator. It's like a rule in a function! Let's apply that rule to the numerator. What are the numerators in each fraction?**

Students *x* and 2.

Teacher **Let's solve for *x* using the same rule. How could we solve for *x*?**

Students Figure out what you can divide by 4 to get 2.

Teacher **Using the same rule as the denominator, *x* would be 8. 8 divided by 4 equals 2. What's *x*?**

Students 8.

Teacher **Let's check. Does the rule work with the relationship between the numerators?**

Students Yes.

Teacher **Does the rule work with the relationship between the denominators?**

Students Yes.

Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators or denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**What's one way we solved for an unknown in a proportion?**
Students	We multiplied the first denominator by the first numerator and the second numerator. Then, we multiplied the second denominator by the first numerator and the second numerator. Then, we divided by a coefficient to solve for the unknown.
Teacher	**What's another way we solved for an unknown in a proportion?**
Students	We determined the rule of the relationship between the denominators and applied that rule to determine the numerators.

(3) Representing Rates

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching

ROUTINE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on rates. A rate is a ratio that compares two different units. What's a rate?**
Students	A ratio that compares two different units.
Teacher	**Units might be *miles* a car can drive per *gallon* of gas. *Miles* and *gallons* are the two different units. Can you share two other units that might be used to show a rate?**
Students	(Shares example.)
Teacher	**Another example might be *dollars* per *package* of strawberries. *Dollars* and *packages* are the two different units. Look at this problem.** (Show problem.)
Teacher	**Often, when solving problems about rate, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**When determining the rate, we'll interpret each fraction in a proportion in the same way. We'll use the same unit for the numerator. We'll then use the other unit for the denominator. How will we think of the two different units with the numerator and denominator?**
Students	The numerator will represent one unit. The denominator will represent the other unit.
Teacher	**In this problem, we have to figure out the unit rate. That is, what is the value for 1 of __** (unit). **What is the unit rate?**
Students	The value for 1 of something.
Teacher	**The unit rate is the value for 1 of __** (unit)**. We can use a proportion to determine the unit rate. 1 divided by *x* can be used in the proportion to represent the unit rate. What can be used to represent the unit rate?**
Students	1 divided by *x*.
Teacher	**We have to determine 1 of *x* in this proportion. We can do this by isolating the unknown or *x*. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	__.

Teacher	**The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out.)
Teacher	**Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the second fraction. Let's write __.** (Write.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.

Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. That's the unit rate. One *x* equals __. What's the unit rate?**
Students	__.
Teacher	**Now, there is another way to solve for an unknown to determine the unit rate. Where is the unknown in this problem?**
Students	Numerator/denominator.
Teacher	**The unknown is in the numerator/denominator, so look at the denominators/numerators. Look at __** (first denominator/numerator) **and __** (second denominator/numerator)**. What do you notice about the relationship between __ and __?**
Students	(Describes relationship.)
Teacher	**Yes! I see that if you multiply/divide by __ with the first denominator/numerator, that equals the second denominator/numerator. It's like a rule in a function! Let's apply that rule to the numerator/denominator. What's the numerator/denominator in each fraction?**
Students	__/*x* and *x*/__.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Multiply/divide.
Teacher	**Using the same rule as the denominator/numerator, *x* would be __. What's *x*?**
Students	__.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators/denominators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's the unit rate?**
Students	The value for 1 of something.

Teacher	**How did we determine the unit rate for an unknown in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown. Or, we determined the rule between numerators and applied that rule to the denominators.

Example

$$\frac{7}{301} = \frac{1}{x}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on rates. A rate is a ratio that compares two different units. What's a rate?**
Students	A ratio that compares two different units.
Teacher	**Units might be *miles* a car can drive per *gallon* of gas. *Miles* and *gallons* are the two different units. Can you share two other units that might be used to show a rate?**
Students	(Shares example.)
Teacher	**Another example might be *dollars* per *package* of strawberries. *Dollars* and *packages* are the two different units. Look at this problem.** (Show problem.)
Teacher	**Often, when solving problems about rate, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**When determining the rate, we'll interpret each fraction in a proportion in the same way. We'll use the same unit for the numerator. We'll then use the other unit for the denominator. How will we think of the two different units with the numerator and denominator?**
Students	The numerator will represent one unit. The denominator will represent the other unit.
Teacher	**In this problem, we have to figure out the unit rate. That is, what is the value for 1 of *x*. What is the unit rate?**
Students	The value for 1 of something.
Teacher	**The unit rate is the value for 1 of *x*. We can use a proportion to determine the unit rate. 1 divided by *x* can be used in the proportion to represent the unit rate. What can be used to represent the unit rate?**
Students	1 divided by *x*.
Teacher	**We have to determine 1 of *x* in this proportion. We can do this by isolating the unknown or *x*. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will need to multiply and divide. What will we do?**
Students	Multiply and divide.

Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	301.
Teacher	**The denominator of the first fraction is 301. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 301.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 301 and have a denominator of 301, 301 divided by 301 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.** (Cross out 301.)
Teacher	**Now, multiply the second numerator by 301.** (Write.) **What's the product of 1 times 301?**
Students	301.
Teacher	**We now have a numerator of 301 in the second fraction. Let's write 301.** (Write 301.)
Teacher	**Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by *x*.** (Write × *x*.) **What's the product of 7 times *x*?**
Students	7*x*.
Teacher	**We now have a numerator of 7*x* in the first fraction. Let's write 7*x*.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write × *x*.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out *x*.)
Teacher	**Using multiplication, we've changed our proportion to the equation 7*x* equals 301. What's the equation?**
Students	7*x* equals 301.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.

Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	7.
Teacher	**Let's divide each side of the equation by 7. Whatever we do to one side of the equal sign we also have to do to the other. What's 301 divided by 7?**
Students	43. (Write.)
Teacher	**So, the variable equals 43. That's the unit rate. One *x* equals 43. What's the unit rate?**
Students	43.
Teacher	**Now, there is another way to solve for an unknown. Where is the unknown in this problem?**
Students	Denominator.
Teacher	**The unknown is in the denominator, so look at the numerators. What do you notice about the relationship between 7 and 1?**
Students	If you divide 7 by 7, that equals 1.
Teacher	**Yes! I see that if you divide 7 by 7, that equals 1. It's like a rule in a function! Let's apply that rule to the denominator. What's the denominator in the first fraction?**
Students	301.
Teacher	**Let's solve for *x* using the same rule. How could we solve for *x*?**
Students	Divide by 7.
Teacher	**Using the same rule as the numerator, divide 301 by 7. What's *x*?**
Students	43.
Teacher	**Let's check. Does the rule work with the relationship between the numerators?**
Students	Yes.
Teacher	**Does the rule work with the relationship between the denominators?**
Students	Yes.
Teacher	**So, another way to solve for an unknown is to determine the rule between the numerators and use that to solve for *x*. Which method do you prefer?**
Students	(Explains preferred method.)
Teacher	**Let's review. What's the unit rate?**
Students	The value for 1 of something.
Teacher	**How did we determine the unit rate for an unknown in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown. Or, we determined the rule between numerators and applied that rule to the denominators.

(4) Representing Percentages

Routine

Materials:

- Module 21 Problem Sets
- Module 21 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 Blocks

ROUTINE WITH BASE-10 BLOCKS

Teacher	**Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**We can show percentages in different ways. Today, let's use these Base-10 blocks.** (Show manipulatives.)
Teacher	**Look at this flat. How many units are in this flat?**
Students	100.
Teacher	**A percentage is how many per hundred. So, if we have 100 cubes in the flat, the flat can represent the hundred. Let's leave the flat on the table. Now, let's focus on the percentage. Look at this problem.** (Show problem.)
Teacher	**What's the percentage?**
Students	__%.
Teacher	**In this problem, the percentage is __. So, we can show this percentage by showing __ Base-10 blocks on top of the flat.** (Show percentage.)
Teacher	**So, what percentage did we show?**
Students	__%.
Teacher	**Is __ less or greater than 50%?**
Students	Less/greater.
Teacher	**Is __ less or greater than 100?**
Students	Less/greater.
Teacher	**You can use these blocks to help you understand the value of the percentage. Let's review.**
Teacher	**What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**How can you use Base-10 blocks to show a percentage?**
Students	Show the hundred flat. Then place the percentage, using Base-10 blocks, on top of the flat.

ROUTINE WITHOUT MANIPULATIVES

Teacher **Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**

Students A rate of an amount per hundred.

(Show problem.)

Teacher **When determining the percentage of something, we use a proportion. What's a proportion?**

Students An equation with two equal ratios.

Teacher **So, in our proportion, we want to determine the percentage of a fraction or ratio. We can show this as __** (fraction) **is equal to *x* divided by 100. How can we represent the percentage?**

Students *x* divided by 100.

Teacher **In this problem, we have to figure out the percentage. That is, what is the value for *x* per 100. We can do this by isolating the unknown. Another word for unknown is variable. Say that with me.**

Students Variable.

Teacher **In this proportion, if we want to isolate the variable, we will multiply and divide. What will we do?**

Students Multiply and divide.

Teacher **First, let's multiply. What's the denominator of the first fraction?**

Students __.

Teacher **The denominator of the first fraction is __. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**

Students Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.

Teacher **Let's multiply the first denominator times the first numerator.** (Write.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by __** (first denominator) **and have a denominator of __** (first denominator)**, __ divided by __ equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**

Students Canceling or cancellation.

Teacher **I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.**

(Cross out.)

Teacher **Now, multiply the second numerator by __** (first denominator)**.** (Write.) **What's the product of __ times __?**

Students __.

Teacher **We now have a numerator of __ in the second fraction. Let's write __.** (Write.)

Teacher **Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**

Students	Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first numerator by __** (second denominator)**.** (Write.) **What's the product of __ times __?**
Students	__.
Teacher	**We now have a numerator of __ in the first fraction. Let's write __.** (Write.)
Teacher	**Now, let's multiply the second denominator times the second numerator.** (Write.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.** (Cross out.)
Teacher	**Using multiplication, we've changed our proportion to the equation __ equals __. What's the equation?**
Students	__ equals __.
Teacher	**Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**
Students	It's the constant multiplied by a variable.
Teacher	**A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**
Students	Divide each side of the equation by the coefficient.
Teacher	**What's the coefficient?**
Students	__.
Teacher	**Let's divide each side of the equation by __** (coefficient)**. Whatever we do to one side of the equal sign we also have to do to the other. What's __ divided by __?**
Students	__. (Write.)
Teacher	**So, the variable equals __. That's the percentage. What's the percentage?**
Students	__.
Teacher	**Let's review. What's a percentage?**
Students	A rate of an amount per hundred.
Teacher	**How did we determine the percentage in a proportion?**
Students	We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown.

Example

$$\frac{32}{40} = \frac{x}{100}$$

EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Today, let's work on percentages. A percentage is just a rate that tells how many of something per hundred. What's a percentage?**
Students	A rate of an amount per hundred.
	(Show problem.)
Teacher	**When determining the percentage of something, we use a proportion. What's a proportion?**
Students	An equation with two equal ratios.
Teacher	**So, in our proportion, we want to determine the percentage of a fraction or ratio. We can show this as $\frac{32}{40}$ is equal to *x* divided by 100. How can we represent the percentage?**
Students	*x* divided by 100.
Teacher	**In this problem, we have to figure out the percentage. That is, what is the value for *x* per 100. We can do this by isolating the unknown. Another word for unknown is variable. Say that with me.**
Students	Variable.
Teacher	**In this proportion, if we want to isolate the variable, we will multiply and divide. What will we do?**
Students	Multiply and divide.
Teacher	**First, let's multiply. What's the denominator of the first fraction?**
Students	40.
Teacher	**The denominator of the first fraction is 40. We multiply the denominator of the first fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**
Students	Multiply the first denominator by the numerator of the first fraction and the numerator of the second fraction.
Teacher	**Let's multiply the first denominator times the first numerator.** (Write × 40.) **If we do this, the first fraction becomes a whole number. This works because if we multiply the numerator by 40 and have a denominator of 40, 40 divided by 40 equals 1. This is often called canceling or cancellation. What can we do when the numerator and denominator are the same?**
Students	Canceling or cancellation.
Teacher	**I like to show the canceling by crossing out the first denominator and the multiplied amount in the first numerator.**
	(Cross out 40.)
Teacher	**Now, multiply the second numerator by 40.** (Write × 40.) **What's the product of *x* times 40?**
Students	40*x*.
Teacher	**We now have a numerator of 40*x* in the second fraction. Let's write 40*x*.**

(Write 40*x*.)

Teacher **Now we do the same thing with the second denominator. We multiply the denominator of the second fraction by the numerator of the first fraction and the numerator of the second fraction. What should we do?**

Students Multiply the second denominator by the numerator of the first fraction and the numerator of the second fraction.

Teacher **Let's multiply the first numerator by 100.** (Write × 100.) **What's the product of 32 times 100?**

Students 3,200.

Teacher **We now have a numerator of 3,200 in the first fraction. Let's write 3,200.** (Write 3,200.)

Teacher **Now, let's multiply the second denominator times the second numerator.** (Write × 100.) **If we do this, the second fraction becomes a whole number. What can we do when the numerator and denominator are the same?**

Students Canceling or cancellation.

Teacher **I like to show the canceling by crossing out the second denominator and the multiplied amount in the second numerator.**
(Cross out 100.)

Teacher **Using multiplication, we've changed our proportion to the equation 3,200 equals 40*x*. What's the equation?**

Students 3,200 equals 40*x*.

Teacher **Now, we solve for the unknown. To determine the value of the unknown, we divide by the coefficient. What's a coefficient?**

Students It's the constant multiplied by a variable.

Teacher **A coefficient tells us the number of groups of the unknown. If we divide each side of the equation by the coefficient, we will isolate the variable. What do we need to do?**

Students Divide each side of the equation by the coefficient.

Teacher **What's the coefficient?**

Students 40.

Teacher **Let's divide each side of the equation by 40. Whatever we do to one side of the equal sign we also have to do to the other. What's 3,200 divided by 40?**

Students 80.
(Write.)

Teacher **So, the variable equals 80. That's the percentage. What's the percentage?**

Students 80%.

Teacher **Let's review. What's a percentage?**

Students A rate of an amount per hundred.

Teacher **How did we determine the percentage in a proportion?**

Students We first multiplied each denominator times each numerator. Then, we divided by the coefficient to solve for the unknown.

D. Problems for Use During Instruction

See Module 21 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 21 Vocabulary Cards.

Developed by:
Sarah R. Powell (srpowell@austin.utexas.edu)
Katherine A. Berry (kberry@austin.utexas.edu)

Module 21:

Ratios, Proportions, Rates, and Percentages

Problem Sets

A. Ratios (30)
B. Proportions (40)
C. Unit rates (20)
D. Percentages (20)
E. Determining percentages (10)

A.

2 : 3

A.

1 : 4

A.

1 : 7

A.

1 : 2

A.

6 : 6

A.

2 : 1

A.

7 : 3

A.

2 : 9

A.

3 : 4

A.

5 : 8

A.

1 : 5

A.

9 : 8

A.

4 : 6

A.

$$7:8$$

A.

1 : 2

A.

15 : 6

A.

10 : 1

A.

11 : 3

A.

2 : 30

A.

3 : 20

A.

2 : 15

A.

6 : 32

A.

5:11

A.

4 : 24

A.

1 : 20

A.

3 : 15

A.

9 : 90

A.

50 : 150

A.

10 : 300

A. 20 : 600

B.

$$\frac{?}{4} = \frac{3}{12}$$

B.

$$\frac{x}{6} = \frac{15}{18}$$

B.

$$\frac{?}{3} = \frac{16}{24}$$

B.

$$\frac{x}{6} = \frac{2}{12}$$

B.

$$\frac{?}{20} = \frac{4}{5}$$

B.

$$\frac{x}{3} = \frac{9}{12}$$

B.

$$\frac{?}{8} = \frac{1}{2}$$

B.

$$\frac{x}{16} = \frac{5}{8}$$

B.

$$\frac{?}{100} = \frac{1}{4}$$

B.

$$\frac{x}{100} = \frac{1}{10}$$

B.

$$\frac{1}{?} = \frac{3}{12}$$

B.

$$\frac{2}{x} = \frac{12}{24}$$

B.

$$\frac{1}{?} = \frac{10}{30}$$

B.

$$\frac{3}{x} = \frac{6}{20}$$

B.

$$\frac{1}{?} = \frac{2}{6}$$

B.

$$\frac{28}{x} = \frac{4}{6}$$

B.

$$\frac{36}{?} = \frac{4}{5}$$

B.

$$\frac{9}{x} = \frac{1}{6}$$

B.

$$\frac{25}{?} = \frac{1}{4}$$

B.

$$\frac{20}{x} = \frac{2}{10}$$

B.

$$\frac{1}{4} = \frac{?}{12}$$

B.

$$\frac{3}{5} = \frac{x}{15}$$

B.

$$\frac{4}{5} = \frac{?}{35}$$

B.

$$\frac{2}{6} = \frac{x}{48}$$

B.

$$\frac{1}{4} = \frac{?}{8}$$

B.

$$\frac{16}{20} = \frac{x}{15}$$

B.

$$\frac{21}{35} = \frac{?}{5}$$

B.

$$\frac{4}{40} = \frac{x}{10}$$

B.

$$\frac{40}{100} = \frac{?}{25}$$

B.

$$\frac{100}{125} = \frac{x}{25}$$

B.

$$\frac{4}{8} = \frac{28}{?}$$

B.

$$\frac{1}{4} = \frac{3}{x}$$

B.

$$\frac{1}{4} = \frac{5}{?}$$

B.

$$\frac{2}{3} = \frac{12}{x}$$

B.

$$\frac{6}{12} = \frac{18}{?}$$

B.

$$\frac{16}{20} = \frac{4}{x}$$

B.

$$\frac{8}{80} = \frac{1}{?}$$

B.

$$\frac{8}{16} = \frac{3}{x}$$

B.

$$\frac{200}{500} = \frac{100}{?}$$

B.

$$\frac{50}{100} = \frac{5}{x}$$

c.

$$\frac{5}{45} = \frac{1}{x}$$

c.

$$\frac{8}{80} = \frac{1}{x}$$

c.

$$\frac{3}{27} = \frac{1}{x}$$

c.

$$\frac{8}{56} = \frac{1}{x}$$

c.

$$\frac{15}{45} = \frac{1}{x}$$

c.

$$\frac{12}{144} = \frac{1}{x}$$

c.

$$\frac{7}{182} = \frac{1}{x}$$

c.

$$\frac{6}{210} = \frac{1}{x}$$

c.

$$\frac{13}{195} = \frac{1}{x}$$

c.

$$\frac{8}{400} = \frac{1}{x}$$

c.

$$\frac{16}{64} = \frac{1}{x}$$

c.

$$\frac{3}{165} = \frac{1}{x}$$

c.

$$\frac{7}{539} = \frac{1}{x}$$

c.

$$\frac{5}{60} = \frac{1}{x}$$

c.

$$\frac{2}{166} = \frac{1}{x}$$

c.

$$\frac{4}{76} = \frac{1}{x}$$

c.

$$\frac{25}{300} = \frac{1}{x}$$

c.

$$\frac{12}{150} = \frac{1}{x}$$

c.

$$\frac{7}{427} = \frac{1}{x}$$

c.

$$\frac{4}{932} = \frac{1}{x}$$

D.

50%

D.

12%

D.

75%

D.

24%

D.

96%

D.

37%

D.

8%

D.

42%

D.

62%

D.

79%

E.

$$\frac{32}{40} = \frac{x}{100}$$

E.

$$\frac{4}{5} = \frac{x}{100}$$

E.

$$\frac{18}{20} = \frac{x}{100}$$

E.

$$\frac{4}{50} = \frac{x}{100}$$

E.

$$\frac{6}{25} = \frac{x}{100}$$

E.

$$\frac{8}{16} = \frac{x}{100}$$

E.

$$\frac{28}{70} = \frac{x}{100}$$

E.

$$\frac{1}{5} = \frac{x}{100}$$

E.

$$\frac{14}{20} = \frac{x}{100}$$

E.

$$\frac{18}{30} = \frac{x}{100}$$

E.

$$\frac{7}{35} = \frac{x}{100}$$

E.

$$\frac{22}{44} = \frac{x}{100}$$

E.

$$\frac{9}{12} = \frac{x}{100}$$

E.

$$\frac{12}{80} = \frac{x}{100}$$

E.

$$\frac{24}{40} = \frac{x}{100}$$

E.

$$\frac{3}{4} = \frac{x}{100}$$

E.

$$\frac{1}{2} = \frac{x}{100}$$

E.

$$\frac{11}{25} = \frac{x}{100}$$

E.

$$\frac{17}{25} = \frac{x}{100}$$

E.

$$\frac{3}{5} = \frac{x}{100}$$

Module 21:

Ratios, Proportions, Rates, and Percentages

Vocabulary Cards

coefficient
constant
denominator
equal sign
equivalent fractions
equivalent ratios
fraction
improper fraction
least common multiple
like fractions
lowest terms
mixed number
multiple
numerator
percentage
proper fraction
proportion
rate
ratio
unit rate
unlike fractions
variable

coefficient

A number that is multiplied by a variable.

$$5x + 9 = 24$$

5 is a coefficient

---

constant

A term that does not change; a number on its own.

$$5x + 9 = 24$$

9 and 24 are constants

denominator

The term in a fraction that tells the number of equal parts in a whole.

2 / 3 $\quad \frac{2}{3}$ **In these fractions, 3 is the denominator.**

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equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

12 + 8 = 20

= is the equal sign

equivalent fractions

Fractions that have different numerators and denominators that represent the same value or proportion of the whole.

$$\frac{1}{4} = \frac{2}{8} \qquad \frac{2}{3} = \frac{8}{12}$$

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equivalent ratios

Ratios that have the same fractional number, value, or measure.

1 : 7 = 2 : 14 **2 : 5 = 4 : 10**

fraction

A number representing part of a whole or set.

$$\frac{3}{6} \qquad \frac{10}{12} \qquad \frac{8}{3}$$

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improper fraction

Any fraction in which the numerator is greater than the denominator.

$$\frac{9}{4} \qquad \frac{17}{12} \qquad \frac{10}{3}$$

least common multiple

The common multiple with the least value.

6: 6, 12, 18, 24, 30
8: 8, 16, 24, 32, 40

With multiples of 6 and 8, the least common multiple is 24.

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like fractions

Fractions that have the same denominator.

$$\frac{1}{4} \qquad \frac{2}{4} \qquad \frac{3}{4}$$

lowest terms

A fraction is reduced to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.

$$\frac{2}{8} = \frac{1}{4}$$

lowest terms

$$\frac{3}{9} = \frac{1}{3}$$

lowest terms

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mixed number

A whole number and a fraction combined.

$$1\frac{1}{6} \qquad 4\frac{5}{12} \qquad 12\frac{4}{3}$$

multiple

The product of a number and any integer.

4: 4, 8, 12, 16, 20

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numerator

The term in a fraction that tells how many parts of a fraction.

2 / 3

$$\frac{2}{3}$$

In these fractions, 2 is the numerator.

percentage

A rate of an amount per hundred.

$$\frac{3}{4} = \frac{x}{100} = 75\%$$

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proper fraction

A fraction where the numerator is less than the denominator.

$$\frac{3}{4} \qquad \frac{5}{6} \qquad \frac{8}{21}$$

proportion

An equation that states that two ratios are equal.

$$\frac{2}{3} = \frac{4}{6} \qquad \frac{5}{15} = \frac{1}{3}$$

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rate

A comparison of two quantities that have different units of measure.

$$\frac{25 \text{ feet}}{3 \text{ hours}} \qquad \frac{4 \text{ words}}{2 \text{ minutes}}$$

rate rate

ratio

A comparison of two quantities that have the same unit of measure.

The ratio of ♀ to ♂ is 4 : 3 or 4/3.

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unit rate

A ratio that is written as a number to one.

$$\frac{60 \text{ miles}}{3 \text{ hours}} = \frac{20 \text{ miles}}{1 \text{ hour}}$$

rate — unit rate

unlike fractions

Fractions that have different denominators.

$$\frac{1}{2} \qquad \frac{1}{3} \qquad \frac{1}{7}$$

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variable

A symbol for an unknown value, which is usually represented by a letter.

$$5x + 9 = 24$$

x is a variable