INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 18

Addition and Subtraction of Integers

Module 18: Addition and Subtraction of Integers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
absolute value	The distance of a number from 0 on a number line.
addend	Any numbers added together.
difference	The result of subtracting one number from another number.
integer	A positive or negative whole number.
minuend	The number from which another number is subtracted.
negative number	Any number less than 0.
number line	A straight line with numbers placed at equal intervals along its length.
opposites	Two numbers that are equal distance from 0 on a number line.
positive number	Any number greater than 0.
subtrahend	The number to be subtracted.
sum	The result of adding two or more numbers or the total number when you combine sets.
zero pair	A pair of numbers with a sum of 0.

B. Background Information

In this module, we focus on addition and subtraction of integers. An integer is a positive or negative whole number. We use the following different models to help students understand addition and subtraction of integers:

(1) Addition with a Number Line
(2) Subtraction with a Number Line
(3) Addition with Two-Color Counters
(4) Subtraction with Two-Color Counters
(5) Addition with a Positive and Negative Mat with Cubes
(6) Subtraction with a Positive and Negative Mat with Cubes

When referring to integers, be sure to emphasize that numbers without a negative symbol (-) are assumed positive. So:

7 is “positive seven” or “seven.”
-7 is “negative seven.”

Be sure to use the negative symbol (-), instead of a minus sign (–), for representing negative numbers.

Emphasize *zero pairs* when teaching integers. A zero pair is a pair of numbers with a sum of 0. So, -7 + 7 = 0.

C. Routines and Examples

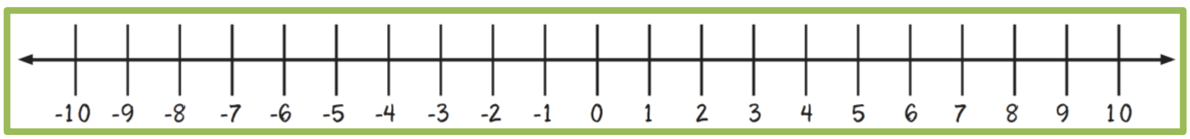
(1) Addition with a Number Line

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



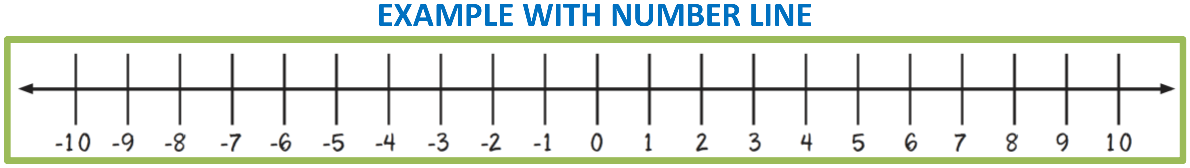
Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this number line.** (Show number line.) (Show problem.)

Teacher	**What numbers are we adding?**
Students	__ plus __.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	__.
Teacher	**Let's place the duck on the number line at the first addend. When adding, we'll place duck so it is facing the increasing numbers on the number line.** (Place duck on first addend. Make sure duck is facing increasing number on number line.)
Teacher	**Now, let's add. What number do we add?**
Students	__.
Teacher	**If the second addend is positive, we move forward on the number line. What do we do if the second addend is positive?**
Students	Move forward on the number line.
Teacher	**If the second addend is negative, we move backward on the number line. What do we do if the second addend is negative?**
Students	Move backward on the number line.
Teacher	**So, which direction should we move?**
Students	Forward/backward.
Teacher	**Because the second addend is positive/negative, we move forward/backward __ spaces. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**So, our duck shows the sum. What's __ plus __?**
Students	__.
Teacher	**Yes. __ plus __ equals __. Using this number line helps you understand what it means to add integers. How can you use the number line to add integers?**
Students	Start at the first addend. If the second addend is positive, move forward on the number line. If the second addend is negative, move backward on the number line.

Example

-3 + 5

EXAMPLE WITH NUMBER LINE



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.

Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we adding?**
Students	-3 plus 5.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	-3.
Teacher	**Let's place the duck on the number line at the first addend. When adding, we'll place the duck so it is facing the increasing numbers on the number line.** (Place duck on -3.)
Teacher	**Now, let's add. What number do we add?**
Students	5.
Teacher	**If the second addend is positive, we move forward on the number line. What do we do if the second addend is positive?**
Students	Move forward on the number line.
Teacher	**If the second addend is negative, we move backward on the number line. What do we do if the second addend is negative?**
Students	Move backward on the number line.
Teacher	**So, which direction should we move?**
Students	Forward.
Teacher	**Because the second addend is positive, we move forward 5 spaces. Let's do that together. Count with me.**
Students	1, 2, 3, 4, 5.
Teacher	**So, our duck shows the sum. What's -3 plus 5?**
Students	2.
Teacher	**Yes. -3 plus 5 equals 2. Using this number line helps you understand what it means to add integers. How can you use the number line to add integers?**
Students	Start at the first addend. If the second addend is positive, move forward on the number line. If the second addend is negative, move backward on the number line.

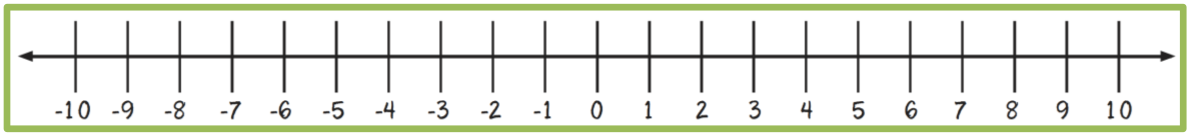
(2) Subtraction with a Number Line

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



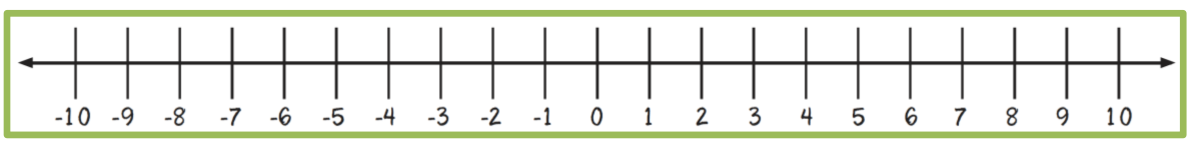
Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	__ minus __.
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	__.
Teacher	**Let's place the duck on the number line at the minuend. When subtracting, we'll place the duck so it is facing the decreasing numbers on the number line.** (Place duck on minuend. Place duck facing the decreasing numbers on the number line.)
Teacher	**Now, let's subtract. What number do we subtract? What's the subtrahend?**
Students	__.

Teacher	**If the subtrahend is positive, we move the duck forward on the number line from where the duck is facing. What do we do if the subtrahend is positive?**
Students	Move forward on the number line from where the duck is facing.
Teacher	**If the subtrahend is negative, we move backward on the number line from where the duck is facing. What do we do if the subtrahend is negative?**
Students	Move backward on the number line from where the duck is facing.
Teacher	**So, which direction should we move?**
Students	Forward/backward.
Teacher	**Because the subtrahend is positive/negative, we move forward/backward __ spaces. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**So, our duck shows the difference. What's __ minus __?**
Students	__.
Teacher	**Yes. __ minus __ equals __. Using this number line helps you understand what it means to subtract integers. How can you use the number line to subtract integers?**
Students	Start at the minuend with the duck facing the decreasing numbers on the number line. If the subtrahend is positive, move the duck forward from its position. If the subtrahend is negative, move the duck backward from its position.

Example

4 – (-3)

EXAMPLE WITH NUMBER LINE



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.

Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	4 minus (-3).
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	4.
Teacher	**Let's place the duck on the number line at the minuend. When subtracting, we'll place the duck so it is facing the decreasing numbers on the number line.** (Place duck on minuend. Place duck facing the decreasing numbers on the number line.)
Teacher	**Now, let's subtract. What number do we subtract? What's the subtrahend?**
Students	-3.
Teacher	**If the subtrahend is positive, we move the duck forward on the number line from where the duck is facing. What do we do if the subtrahend is positive?**
Students	Move forward on the number line from where the duck is facing.
Teacher	**If the subtrahend is negative, we move backward on the number line from where the duck is facing. What do we do if the subtrahend is negative?**
Students	Move backward on the number line from where the duck is facing.
Teacher	**So, which direction should we move?**
Students	Backward.
Teacher	**Because the subtrahend is negative, we move backward 3 spaces. Let's do that together. Count with me.**
Students	1, 2, 3.
Teacher	**So, our duck shows the difference. What's 4 minus -3?**
Students	7.
Teacher	**Yes. 4 minus -3 equals 7. Using this number line helps you understand what it means to subtract integers. How can you use the number line to subtract integers?**
Students	Start at the minuend with the duck facing the decreasing numbers on the number line. If the subtrahend is positive, move the duck forward from its position. If the subtrahend is negative, move the duck backward from its position.

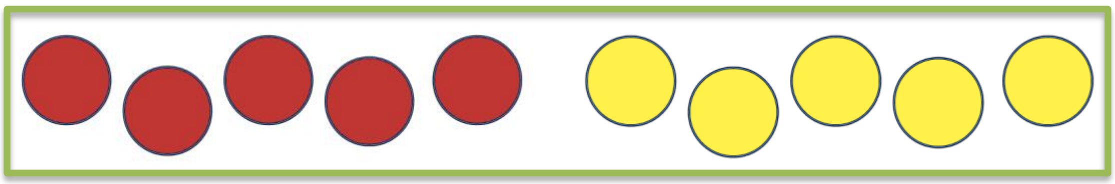
(3) Addition with Two-Color Counters

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let’s add integers. An integer is a positive or negative whole number. What’s an integer?**
Students	A positive or negative whole number.
Teacher	**Let’s think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn’t have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let’s work on adding with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**
Students	Negative. (Show problem.)
Teacher	**What numbers are we adding?**
Students	__ plus __.
Teacher	**So, let’s start at the first addend. What’s the first addend?**

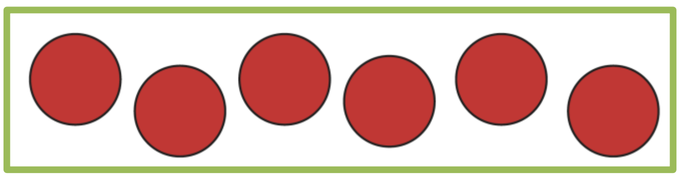
Students	__.	
Teacher	**Let's show the first addend with the two-color counters. How do we show __** (first addend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we'll show __ yellow/red counters.** (Show counters.)	
Teacher	**Now, let's add. What number do we add?**	
Students	__.	
Teacher	**Let's add the second addend to the first. How do we show __** (second addend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we'll show __ yellow/red counters.** (Show counters.)	
Teacher	**Let's add the counters of the second addend to the counters of the first addend.** (Add all counters together.)	
Teacher	**Are the counters all the same color?**	
Students	*OPTION 1:*	Yes!
	OPTION 2:	No!
Teacher	***OPTION 2:***	**When we have a mix of yellow and red counters, we'll make zero pairs until we only have all positive counters or all negative counters. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students		0.
Teacher		**Let's make zero pairs. I see a positive and a negative** (place counters side-by-side)**. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. We can remove this pair from our workspace.** (Remove zero pair.)
Teacher		**Can we make another zero pair?** (Continue removing zero pairs until there are no more zero pairs.)
Teacher		**Are the counters all the same color?**
Students		Yes!
Teacher	**All of the counters are the same color. How many counters?**	
Students	__.	
Teacher	**So, what's __ plus __?**	
Students	__.	

Teacher	**__ plus __ equals __. Let’s say that together.**
Students	__ plus __ equals __.
Teacher	**Nice job! Using the two-color counters helps you add integers. How can you use the two-color counters to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the counters are positive or negative. The sum is the remaining counters.

Example

2 + -8

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher	**Let’s add integers. An integer is a positive or negative whole number. What’s an integer?**
Students	A positive or negative whole number.
Teacher	**Let’s think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn’t have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let’s work on adding with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**
Students	Negative. (Show problem.)

Teacher	**What numbers are we adding?**
Students	2 plus -8.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	2
Teacher	**Let's show the first addend with the two-color counters. How do we show 2?**
Students	Show 2 yellow counters.
Teacher	**Yes, we'll show 2 yellow counters.**
	(Show counters.)
Teacher	**Now, let's add. What number do we add?**
Students	-8.
Teacher	**How do we show -8?**
Students	Show 8 red counters.
Teacher	**Yes, we'll show 8 red counters.**
	(Show counters.)
Teacher	**Let's add the counters of the second addend to the counters of the first addend.**
	(Add all counters together.)
Teacher	**Are the counters all the same color?**
Students	No!
Teacher	**When we have a mix of yellow and red counters, we'll make zero pairs until we only have all positive counters or all negative counters. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs. I see a positive and a negative** (place counters side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	Yes!
Teacher	**We can make another zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	No.
Teacher	**All of the counters are the same color. How many counters?**
Students	-6.
Teacher	**So, what's 2 plus -8?**
Students	-6.

Teacher	**2 plus -8 equals -6. Let's say that together.**
Students	2 plus -8 equals -6.
Teacher	**Nice job! Using the two-color counters helps you add integers. How can you use the two-color counters to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the counters are positive or negative. The sum is the remaining counters.

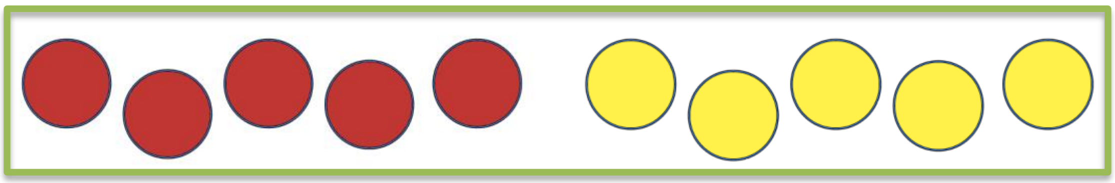
(4) Subtraction with Two-Color Counters

Routine

Materials:

- [Module 18 Problem Sets]
- [Module 18 Vocabulary Cards]
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like two-color counters or multi-colored cubes

ROUTINE WITH TWO-COLOR COUNTERS



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**
Students	Negative. (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	__ minus __.
Teacher	**So, let's start with the minuend. What's the minuend?**

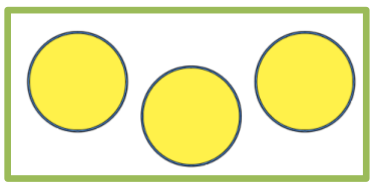
Students	__.	
Teacher	**Let’s show the minuend with the two-color counters. How do we show __** (minuend)**?**	
Students	Show __ yellow/red counters.	
Teacher	**Yes, we’ll show __ yellow/red counters.** (Show counters.)	
Teacher	**Now, let’s subtract. What’s the subtrahend?**	
Students	__.	
Teacher	**We need to subtract __** (subtrahend). **Look at the minuend. Do we have __ positive/negative counters to subtract or take away __** (subtrahend) **counters?**	
Students	*OPTION 1:*	Yes!
	OPTION 2:	No!
Teacher	***OPTION 2:***	**We don’t have enough positive/negative counters to subtract the minuend. We can make zero pairs until we have enough counters to subtract the __** (subtrahend) **counters. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What’s a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What’s +1 and -1?**
Students		0.
Teacher		**Let’s make zero pairs for our workspace. Let’s add one zero pair. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. Let’s bring that zero pair to the workspace.** (Add zero pair.)
Teacher		**Look at the counters. Do we have enough __** (positive/negative) **counters to subtract __** (subtrahend) **counters?**
Students		Yes/no.
Teacher		**We keep making zero pairs until we have enough positive/negative counters to subtract __** (subtrahend) **counters.** (Continue making zero pairs until there are enough positive/negative counters to subtract.)
Teacher	**Now, let’s subtract the subtrahend. That means we’ll subtract or take away __ counters.** (Subtract.)	
Teacher	**How many counters?**	
Students	__.	

Teacher	**So, what's __ minus __?**
Students	__.
Teacher	**__ minus __ equals __. Let's say that together.**
Students	__ minus__ equals __.
Teacher	**Nice job! Using the two-color counters helps you subtract integers. How can you use the two-color counters to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining counters.

Example

-1 – (-4)

EXAMPLE WITH TWO-COLOR COUNTERS



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with these two-color counters.** (Show counters.)
Teacher	**With the two-color counters, the yellow side will represent positive integers. What does the yellow side represent?**
Students	Positive.
Teacher	**And the red side will represent negative integers. What does the red side represent?**

Students	Negative.
	(Show problem.)
Teacher	**What numbers are we subtracting?**
Students	-1 minus -4.
Teacher	**So, let's start with the minuend. What's the minuend?**
Students	-1.
Teacher	**Let's show the minuend with the two-color counters. How do we show -1?**
Students	Show 1 red counter.
Teacher	**Yes, we'll show 1 red counter.**
	(Show counter.)
Teacher	**Now, let's subtract. What's the subtrahend?**
Students	-4.
Teacher	**We need to subtract -4 or 4 red counters. Look at the minuend. Do we have enough negative counters to subtract or take away 4 counters?**
Students	No!
Teacher	**We don't have enough negative counters to subtract the minuend. We can make zero pairs until we have enough counters to subtract the 4 counters. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the counters. Now we have 2 negative counters. Do we have enough negative counters to subtract 4 counters?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the counters. Now we have 3 negative counters. Do we have enough negative counters to subtract 4 counters?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)

Teacher	**Look at the counters. Now we have 4 negative counters. Do we have enough negative counters to subtract 4 counters?**
Students	Yes!
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away 4 red counters.** (Subtract.)
Teacher	**How many counters do we have now?**
Students	3.
Teacher	**So, what's -1 minus -4?**
Students	3.
Teacher	**-1 minus -4 equals 3. Let's say that together.**
Students	-1 minus -4 equals 3.
Teacher	**Nice job! Using the two-color counters helps you subtract integers. How can you use the two-color counters to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining counters.

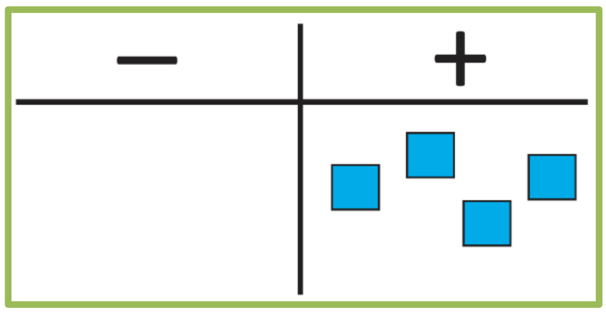
(5) Addition with Positive and Negative Mat with Cubes

Routine

Materials:

- Module 18 Problem Sets
- Module 18 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or paperclips

ROUTINE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side.

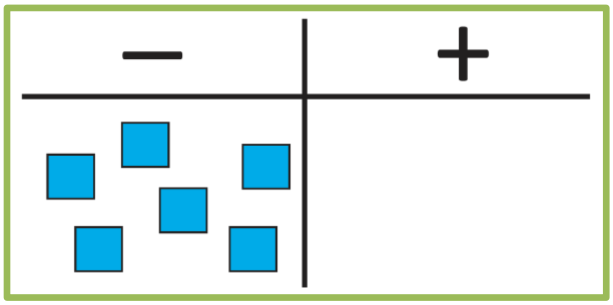
	(Show problem.)	
Teacher	**What numbers are we adding?**	
Students	__ plus __.	
Teacher	**So, let’s start at the first addend. What’s the first addend?**	
Students	__.	
Teacher	**Let’s show the first addend with the cubes. How do we show __** (first addend)**?**	
Students	Show __ cubes on the positive/negative side.	
Teacher	**Yes, we’ll show __ cubes on the positive/negative side of the mat.** (Show cubes.)	
Teacher	**Now, let’s add. What number do we add?**	
Students	__.	
Teacher	**Let’s add the second addend to the first. How do we show __** (second addend)**?**	
Students	Show __ cubes on the positive/negative side.	
Teacher	**Yes, we’ll show __ cubes on the positive/negative side of the mat.** (Show cubes.)	
Teacher	**Are the cubes on the same side of the mat?**	
Students	*OPTION 1:*	Yes!
	OPTION 2:	No!
Teacher	***OPTION 2:***	**When we have cubes on the positive side and negative side, we’ll make zero pairs until we only have all positive cubes or all negative cubes. What we will make?**
Students		Zero pairs.
Teacher		**A zero pair is when you add a positive number and its opposite. The sum is zero. What’s a zero pair?**
Students		When you add a positive number and its opposite. The sum is zero.
Teacher		**So, positive 1 and negative 1 equals 0. What’s +1 and -1?**
Students		0.
Teacher		**Let’s make zero pairs. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**
Students		A zero pair.
Teacher		**We made a zero pair. We can remove this pair from our workspace.** (Remove zero pair.)
Teacher		**Can we make another zero pair?** (Continue removing zero pairs until there are no more zero pairs.)
Teacher	**All of the cubes are on the same side of the mat. How many cubes?**	
Students	__.	
Teacher	**So, what’s __ plus __?**	
Students	__.	
Teacher	**__ plus __ equals __. Let’s say that together.**	

Students	__ plus __ equals __.
Teacher	**Nice job! Using the mat and cubes helps you add integers. How can you use the mat and cubes to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the cubes are positive or negative. The sum is the remaining cubes.

Example

2 + (-7)

EXAMPLE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's add integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on adding with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.

Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side.
	(Show problem.)
Teacher	**What numbers are we adding?**
Students	2 plus -7.
Teacher	**So, let's start at the first addend. What's the first addend?**
Students	2.
Teacher	**Let's show the first addend with the cubes. How do we show 2?**
Students	Show 2 cubes on the positive side.
Teacher	**Yes, we'll show 2 cubes on the positive side of the mat.**
	(Show cubes.)
Teacher	**Now, let's add. What number do we add?**
Students	-7.
Teacher	**Let's add the second addend to the first. How do we show -7?**
Students	Show 7 cubes on the negative side.
Teacher	**Yes, we'll show 7 cubes on the negative side of the mat.**
	(Show cubes.)
Teacher	**Are the cubes on the same side of the mat?**
Students	No!
Teacher	**When we have cubes on both the positive side and negative side, we'll make zero pairs until we only have all positive cubes or all negative cubes. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	Yes!
Teacher	**Let's make another zero pair. I see a positive and a negative** (place cubes side-by-side)**. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. We can remove this pair from our workspace.**
	(Remove zero pair.)
Teacher	**Can we make another zero pair?**
Students	No!
Teacher	**All of the cubes are on the same side of the mat. How many cubes?**

Students	-5.
Teacher	**So, what's 2 plus -7?**
Students	-5.
Teacher	**2 plus -7 equals -5. Let's say that together.**
Students	2 plus -7 equals -5.
Teacher	**Nice job! Using the mat and cubes helps you add integers. How can you use the mat and cubes to add integers?**
Students	You show the first addend. Then, you add the second addend. You create zero pairs until all the cubes are positive or negative. The sum is the remaining cubes.

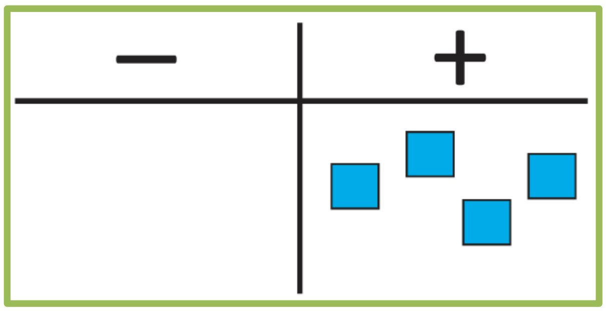
(6) Subtraction with Positive and Negative Mat with Cubes

Routine

Materials:

- [Module 18 Problem Sets]
- [Module 18 Vocabulary Cards]
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes or paperclips

ROUTINE WITH POSITIVE AND NEGATIVE MAT



Teacher	**Let's subtract integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on subtracting with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**

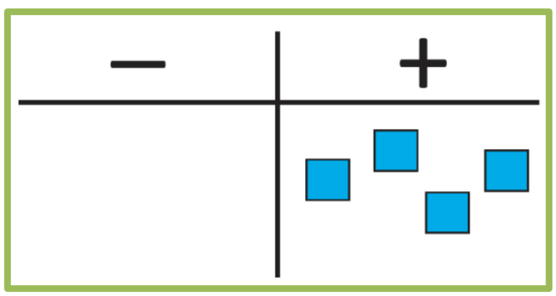
Students	Negative side. (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	__ minus __.
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	__.
Teacher	**Let's show the minuend with the cubes. How do we show __** (minuend)**?**
Students	Show __ cubes on the positive/negative side.
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the mat.** (Show cubes.)
Teacher	**Now, let's subtract. What's the subtrahend?**
Students	__.
Teacher	**We need to subtract how many __ positive/negative cubes?**
Students	__.
Teacher	**So, look at the mat. Do you have enough positive/negative cubes to subtract __ positive/negative cubes?**
Students	*OPTION 1:* Yes. OPTION 2: No.
Teacher	***OPTION 2:*** **We don't have enough positive/negative cubes to subtract the minuend. We can make zero pairs until we have enough cubes to subtract the __** (subtrahend) **cubes. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.** (Add zero pair.)
Teacher	**Look at the cubes. Do we have enough __** (positive/negative) **cubes to subtract __** (subtrahend) **cubes?**
Students	Yes/no.
Teacher	**We keep making zero pairs until we have enough positive/negative cubes to subtract __** (subtrahend) **cubes.** (Continue making zero pairs until there are enough positive/negative cubes to subtract.)
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away __ cubes.** (Subtract.)

Teacher **How many cubes remaining?**
Students __.
Teacher **So, what’s __ minus __?**
Students __.
Teacher **__ minus __ equals __. Let’s say that together.**
Students __ minus__ equals __.
Teacher **Nice job! Using the mat and cubes helps you subtract integers. How can you use the mat and cubes to subtract integers?**
Students You show the minuend. Then, you subtract the subtrahend. If you don’t have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining cubes.

Example

1 – (-3)

EXAMPLE WITH POSITIVE AND NEGATIVE MAT



Teacher **Let’s subtract integers. An integer is a positive or negative whole number. What’s an integer?**
Students A positive or negative whole number.
Teacher **Let’s think about a positive number. How do you know a number is positive?**
Students It has a positive sign or it doesn’t have a sign in front of the number.
Teacher **We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.**
(Draw +.)
Teacher **We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students When there is not a negative sign directly in front of the number.
Teacher **How do you know a number is negative?**
Students It has a negative sign.
Teacher **We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.**
(Draw -.)

Teacher	**Let's work on subtracting with this positive and negative mat and these cubes.** (Show mat and cubes.)
Teacher	**With the mat, we'll place positive integers on the positive side. Where will we place positive integers?**
Students	Positive side.
Teacher	**And we'll place negative integers on the negative side. Where will we place negative integers?**
Students	Negative side. (Show problem.)
Teacher	**What numbers are we subtracting?**
Students	1 minus -3.
Teacher	**So, let's start at the minuend. What's the minuend?**
Students	1.
Teacher	**Let's show the minuend with the cubes. How do we show 1?**
Students	Show 1 cube on the positive side of the mat.
Teacher	**Yes, we'll show 1 cube on the positive side of the mat.** (Show cube.)
Teacher	**Now, let's subtract. What's the subtrahend?**
Students	-3.
Teacher	**We need to subtract how many negative cubes?**
Students	3.
Teacher	**So, look at the mat. Do you have enough negative cubes to subtract 3 negative cubes?**
Students	No.
Teacher	**We don't have enough negative cubes to subtract the minuend. We can make zero pairs until we have enough cubes to subtract the 3 negative cubes. What we will make?**
Students	Zero pairs.
Teacher	**A zero pair is when you add a positive number and its opposite. The sum is zero. What's a zero pair?**
Students	When you add a positive number and its opposite. The sum is zero.
Teacher	**So, positive 1 and negative 1 equals 0. What's +1 and -1?**
Students	0.
Teacher	**Let's make zero pairs for our workspace. Let's add one zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.** (Add zero pair.)
Teacher	**Look at the cubes. Do we have enough negative cubes to subtract 3 cubes?**
Students	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**

	(Add zero pair.)
Teacher	**Look at the cubes. Now we have 2 negative cubes. Do we have enough negative cubes to subtract 3 cubes?**
	No.
Teacher	**Let's add another zero pair. What did we create?**
Students	A zero pair.
Teacher	**We made a zero pair. Let's bring that zero pair to the workspace.**
	(Add zero pair.)
Teacher	**Look at the cubes. Now we have 3 negative cubes. Do we have enough negative cubes to subtract 3 cubes?**
Students	Yes!
Teacher	**Now, let's subtract the subtrahend. That means we'll subtract or take away 3 negative cubes.**
	(Subtract.)
Teacher	**How many cubes remaining?**
Students	4.
Teacher	**So, what's 1 minus -3?**
Students	4.
Teacher	**1 minus -3 equals 4. Let's say that together.**
Students	1 minus -3 equals 4.
Teacher	**Nice job! Using the mat and cubes helps you subtract integers. How can you use the mat and cubes to subtract integers?**
Students	You show the minuend. Then, you subtract the subtrahend. If you don't have enough minuend counters to subtract, you can bring in zero pairs. Then, you subtract. The difference is the remaining cubes.

D. Problems for Use During Instruction

See Module 18 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 18 Vocabulary Cards.

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Module 18:
Addition and Subtraction of Integers

Problem Sets

A.

$3 + (-8)$

A.

$6 + (-4)$

A.

$7 + (-2)$

A.

$5 + (-10)$

A.

$$9 + (-1)$$

A.

$$2 + (\ -5)$$

A.

$$6 + (-3)$$

A.

$7 + (-8)$

A.

$4 + (-2)$

A.

$$17 + (-12)$$

A.

$5 + (-5)$

A.

$$11 + (-15)$$

A.

$8 + (-4)$

A.

$14 + (-8)$

A.

$$7 + (-1)$$

A.

$$12 + (-3)$$

A.

$$1 + (-6)$$

A.

$$4 + (-5)$$

A.

$2 + (-3)$

A.

$$0 + (-9)$$

B.

$$(-1) + 5$$

B.

$$(-3) + 6$$

B.

$$(-9) + 2$$

B.

$(-4) + 3$

B.

$$(-7) + 8$$

B.

$(-8) + 6$

B.

$$(-5) + 4$$

B.

$$(-6) + 10$$

B.

$$(-6) + 3$$

B.

$$(-10) + 5$$

B.

$$(-2) + 2$$

B.

$$(-7) + 8$$

B.

$$(-19) + 13$$

B.

$$(-12) + 3$$

B.

$$(-14) + 6$$

B.

$$(-11) + 8$$

B.

$(-15) + 4$

B.

$$(-5) + 4$$

B.

$(-2) + 0$

B.

$$(-8) + 1$$

c.

$$(-2) + (-3)$$

c.

$$(-6) + (-1)$$

c.

$$(-8) + (-4)$$

c.

$$(-9) + (-9)$$

c.

$$(-5) + (-7)$$

c.

$$(-4) + (-2)$$

c.

$$(-11) + (-6)$$

c.

$$(-3) + (-4)$$

c.

$$(-1) + (-10)$$

c.

$$(-7) + (-12)$$

c.

$$(-9) + (-1)$$

c.

$$(-8) + (-6)$$

c.

$$(-10) + (-9)$$

c.

$$(-2) + (-15)$$

c.

$$(-16) + (-3)$$

c.

$$(-7) + (-14)$$

c.

$$(-12) + (-4)$$

c.

$$(-13) + (-9)$$

c.

$$(-17) + (-4)$$

c.

$$(-16) + (-8)$$

D.

$3 - (-8)$

D.

$6 - (-4)$

D.

$8 - (-2)$

D.

$5 - (-10)$

D.

$$9 - (-1)$$

D.

$$2 - (-5)$$

D.

$6 - (-3)$

D.

7 – (-8)

D.

$4 - (-2)$

D.

$$17 - (-12)$$

D.

$$5 - (-5)$$

D.

$11 - (-15)$

D.

$8 - (-4)$

D.

$14 - (-8)$

D.

$$7 - (-1)$$

D.

$$12 - (-3)$$

D.

$$1 - (-6)$$

D.

$$4 - (-5)$$

D.

$2 - (-3)$

D.

$$0 - (-9)$$

E.

$$(-1) - 5$$

E.

$(-3) - 6$

E.

$$(-9) - 2$$

E.

$$(-4) - 3$$

E.

$$(-7) - 8$$

E.

$$(-8) - 6$$

E.

$$(-5) - 4$$

E.

$$(-6) - 10$$

E.

$$(-6) - 3$$

E.

$$(-10) - 5$$

E.

$$(-2) - 2$$

E.

$$(-7) - 8$$

E.

$(-19) - 13$

E.

$$(-12) - 3$$

E.

$$(-14) - 6$$

E.

$$(-11) - 8$$

E.

$$(-15) - 4$$

E.

$$(-6) - 3$$

E.

$$(-2) - 0$$

E.

$$(-8) - 1$$

F.

$$(-2) - (-3)$$

F.

$$(-6) - (-1)$$

F.

$(-8) - (-4)$

F.

$$(-9) - (-9)$$

F.

$$(-5) - (-7)$$

F.

$$(-4) - (-2)$$

F.

$$(-11) - (-6)$$

F.

$(-3) - (-4)$

F.

$$(-1) - (-10)$$

F.

$$(-7) - (-12)$$

F.

$$(-9) - (-1)$$

F.

$$(-8) - (-6)$$

F.

$(-10) - (-9)$

F.

$$(-2) - (-15)$$

F.

$$(-16) - (-3)$$

F.

$$(-7) - (-14)$$

F.

$(-12) - (-4)$

F.

$$(-13) - (-9)$$

F.

$$(-17) - (-4)$$

F.

$$(-16) - (-8)$$

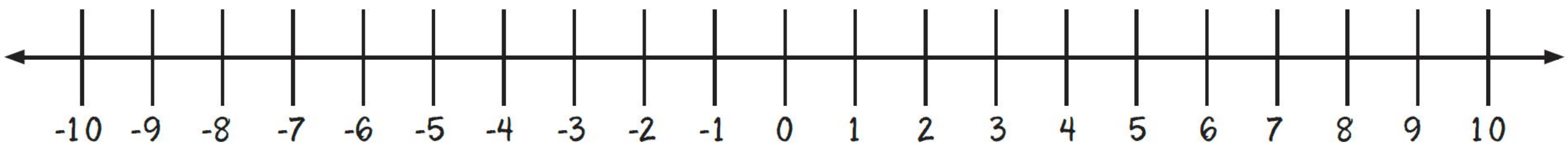
Module 18:
Addition and Subtraction of Integers

Vocabulary Cards

absolute value
addend
difference
integer
minuend
negative number
number line
opposites
positive number
subtrahend
sum
zero pair

absolute value

The distance of a number from 0 on a number line.



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addend

Any numbers added together.

6 + 2 = 8

6 and 2 are addends

difference

The result of subtracting one number from another number.

$$6 - 4 = 2$$

2 is the difference

---

integer

A positive or negative whole number.

-3 -2 -1 1 2 3

minuend

The number from which another number is subtracted.

9 – 4 = 5

9 is the minuend

---

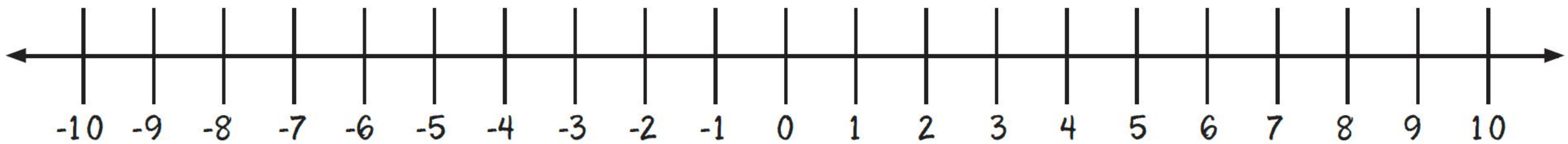
negative number

Any number less than 0.

-3 -2 -1

number line

A straight line with numbers placed at equal intervals along its length.



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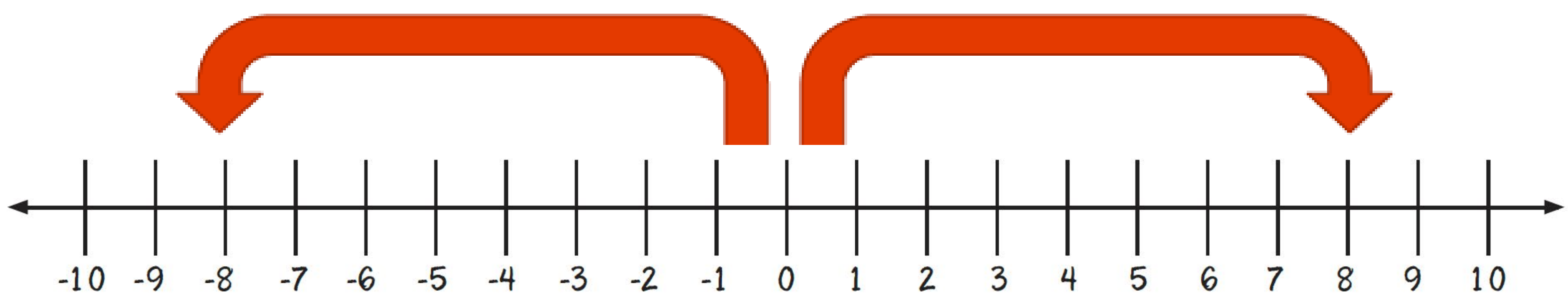
opposites

Two numbers that are equal distance from 0 on a number line.

-8 and 8 are opposites



positive number

Any number greater than 0.

1 2 3

---

subtrahend

The number to be subtracted.

$$9 - 4 = 5$$

4 is the subtrahend

sum

The result of adding two or more numbers or the total number when you combine sets.

$$7 + 2 + 1 = 10$$

10 is the sum

---

zero pair

A pair of numbers with a sum of 0.

$$-7 + 7 = 0$$